Anomalous transport in conical granular piles

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Experiments on $2+1$-dimensional piles of elongated particles are performed. Comparison with previous experiments in $1+1$ dimensions shows that the addition of one extra dimension to the dynamics changes completely the avalanche properties, with the appearance of a characteristic avalanche size. Nevertheless, the time which single grains need to cross the whole pile varies smoothly between several orders of magnitude, from a few seconds to more than 100 hours. This behavior is described by a power-law distribution, signaling the existence of scale invariance in the transport process.

I. INTRODUCTION

Transport phenomena are found at the core of almost every discipline in many-body physics; for example, in nuclear physics, with the motion of neutrons through a fissionable material; in electronics, with the transport of holes and electrons in semiconductors; in astrophysics, with the diffusion of light through stellar atmospheres; and also in plasma physics, biological physics, etc., [1]. The study of granular matter (like sand, powders, and all that) has only very recently become a goal for the physics community (surprisingly, see Ref. [2]) and therefore the understanding of their transport properties will be required in order to explain the (sometimes-strange) properties of these materials [3,4].

Seminal experimental work on transport of individual grains through a granular pile was performed by Christensen *et al.* [5]: a $1+1$-dimensional pile was built in the narrow gap between two parallel plates using elongated rice grains. The time taken by tracer grains to cross the system was measured when the pile was driven out of equilibrium by a constant addition of grains. The probability density of these *transit times* turned out to be, for long times, a decreasing power law with an exponent $\alpha \approx 2.4$, signaling the existence of an anomalous (non-Gaussian) behavior. By means of some theoretical models the transport process was later microscopically understood as a composition of Lévy flights (in the jumps of the grains during an avalanche) and also power-law distributed trapping times [6].

On the other hand, the same rice pile was previously shown to display self-organized criticality (SOC) due to the dissipation introduced in the motion by the elongated shape of the grains, which removes inertial effects [7]. This SOC behavior simply means that the motion of the grains takes place in terms of avalanches, and the distribution of sizes of these avalanches is scale invariant, that is, another power law, achieved without any parameter fine tuning [8]. Interestingly, SOC does not appear when the grains are somewhat round, and therefore more free to roll and accumulate kinetic energy [7].

Additionally, other remarkable properties of this rice-pile system include diverging profile fluctuations with system size [9], and probably a multifractal spectrum of the transit-time time series [10]. More generically, a pile configuration allows one to study the transition between different metastable configurations of a granular system, which occurs when a solid phase becomes unstable and flows like a liquid until it becomes "frozen" again.

Another class of experiments, performed on sandpiles (or with round particles, in general) in $2+1$ dimensions show that in these systems a crossover from small to large avalanches occurs when the system size is increased [11–13]; that is, small piles tend to show avalanches of different sizes, although not very large, but for a big enough system very large avalanches appear in addition, a behavior that is not compatible with SOC [14]. This is in agreement with a different type of experiments performed on rotating drums [15,16].

The extension of rice-pile experiments to $2+1$ dimensions, which is the subject of our research, allows a comparison with the results for sandpiles and raises a series of interesting questions: will the dissipation of the elongated grain motion be enough to suppress inertia and give SOC? If not, which then is the behavior? But even more important, our experiments allow one to explore the transport of individual grains in granular piles, and in general to go deeply into the connection between anomalous transport and SOC, a connection that has not been explored in detail. We find that the behavior of the avalanches in a conical rice pile (with elongated grains) is very similar to what has been observed in conical sandpiles [11], with somewhat small avalanches for small piles and large relaxation oscillations in larger systems. However, the transport properties give totally new information, with a power-law transit-time distribution even in the later case of almost periodic occurrences.

II. EXPERIMENTAL SETUP AND PROCEDURE

The rice pile was built over a wooden (circular) disk supported by a platform which allowed the grains to fall out in any direction when they reached the disk perimeter. Three different disk diameters were employed, $L=10$, 15, and 20 cm, in order to study the effect of the system size. In terms of

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the mean grain length (6.6 mm, see below) this quantity turns out to be $L=15$, 22.5, and 30.

The driving of the pile was carried out by means of a single-seed machine connected to a dc motor through a gearbox. The driving rate was adjusted with a voltage transformer to a value of 43 grains per minute (2 ± 0.4 grains every 2.8 s); this rate is small enough to be considered as a zero driving rate for the system sizes studied (see Ref. [17]) but also allows one to conclude the experiment in a finite time. After being expelled from the machine the grains were directed by a cardboard pipe towards a funnel where at the exit a paper tongue braked the grains and dropped them over the center of the disk with little kinetic energy. Once the pile was built, the distance from the exit of the funnel to the top of the pile was just a few centimeters.

In addition to the described driving procedure, when the pile reached a (statistical) stationary state—where the injected grains were balanced out on average by the coming out grains—tracer grains were added to the top of the pile by hand at a rate of 1 grain every 30 s for the two smallest piles and 1 grain every min for the largest one [18]. These tracer grains were used to compute the transit time, which is the time a grain remains in the pile, since it is added until it comes out. Tracers were marked by a number written on the surface, and were also colored in order to be visually distinguished from the rest of the grains. The addition of each tracer was done at a fixed input time T_{in} (determined by the tracer driving rate), being only necessary then to record the output time T_{out} to get the transit time as $T=T_{out}-T_{in}$. The total number of injected tracers was about 500 for $L=15$ and $L=22.5$ and 700 for $L=30$. After the end of the tracer injection, the experiment and therefore the normal driving of the pile should continue until all tracers come out; however, for practical reasons this could not be achieved and the experiment was stopped when a few tracers were still inside the pile, after more than 100 h. The elimination of these long-living tracers affects the direct calculation of the mean transit time, but not the whole probability distribution, which is a much more interesting quantity, as we will see.

In practice, the grains coming out from the pile were collected by a piece of cardboard and directed at a glass, where the colored tracers were easily recognized by the naked eye. It was then when the output time was recorded, and the glass replaced by another one. Moreover, note that the grains collected in the glass after an avalanche can give a measure of the avalanche size (of course for those avalanches that reach the boundaries). During the addition of the tracers three persons were needed to perform the described tasks, but as long as the number of tracers inside the pile decreases these demands diminish too.

The rice used in all the experiments was "Løse parboiled ris" from Føtex (Denmark). The shape of these grains is elongated, similar to the one of Refs. [7,5], with a length of 6.6 ± 0.9 mm and a width of 1.6 ± 0.6 mm, which gives a length-width ratio of about 4. The mean weight of the grains is 16.9 mg.

III. EXPERIMENTAL RESULTS

The first observation provided by the experiment is a qualitative one: as in sandpiles, the size of the system influences dramatically the behavior. Small piles ($L=15$ and 22.5) do not change their profile very much during time evolution once the pile is built (the total height varies by about 1 cm) and do not display big avalanches either. In contrast, the big pile ($L=30$) shows enormous, catastrophic avalanches, which eject a significant percentage of the grains out of the pile and therefore totally change the profile. In the time interval between these enormous avalanches there are also small avalanches. Actually, this is a similar behavior to what was observed in sandpiles (with round particles) [11]. Therefore the elongation of the rice grains (which was identified as being responsible for SOC in $1+1$-dimensions [7]) does not make enough energy dissipate in three dimensions to give SOC. The addition of one extra degree of freedom to the motion of the grains totally modifies the behavior, in comparison.

To explain the transition in sandpiles when the size of the system is increased, a simple argument was proposed based on the fact that for small piles there is "no room" for the two angles involved in the dynamics (one for the starting of the avalanche and another one for the end) [16]. We find that the situation is not so easy, due to the somewhat rough surface of the pile; indeed, the elongation of the grains gives rise to local bumps of packed grains which provoke fluctuations of the total height of the pile. Moreover, in the regime with large avalanches the measurement of the angle of repose—the angle after an avalanche—makes little sense, since then a large part of the pile has collapsed (like the crater of a volcano) and the conical shape is lost. Rotating-drum experiments could perhaps give more information on this point.

The total mass of dropped grains and the occurrence time of these catastrophic avalanches (in the big pile) was measured. In all cases the number of dropping grains was between about 3000 and 7000, with a mean value of 4600 and a standard deviation of 1100. For comparison, the largest avalanches in the small piles dropped only about 20 or 30 grains, roughly the same value as the largest small avalanches in the big pile. The mean time between big avalanches was 7700 grains (3 h) with a deviation of 2200. The difference with respect to the mean number of dropping grains in big avalanches (4600) corresponds of course to grains coming out in small avalanches.

Let us address now the problem of the transport inside the system. Part of the data for the input times and their corresponding output times for the three experiments are shown in Fig. 1, where transit times are given by the length of the horizontal lines. First, the difference between small and big systems is clear in terms of the number of coincidences in the output times (related to the avalanche size). Moreover, although comparison with Christensen *et al.*'s results [5] is not direct (tracer density is lower there), it is possible to see that their Fig. 2 settles in between our two behaviors. Second, one can observe the great variability of the transit times, broadly distributed from a few seconds to more than 100 h (this last value, not shown in the plot, is in fact a lower bound for the maximum transit time, determined by the duration of the experiment).

Figure 2 displays the probability densities of the transit times measured for each pile. It is apparent once more the
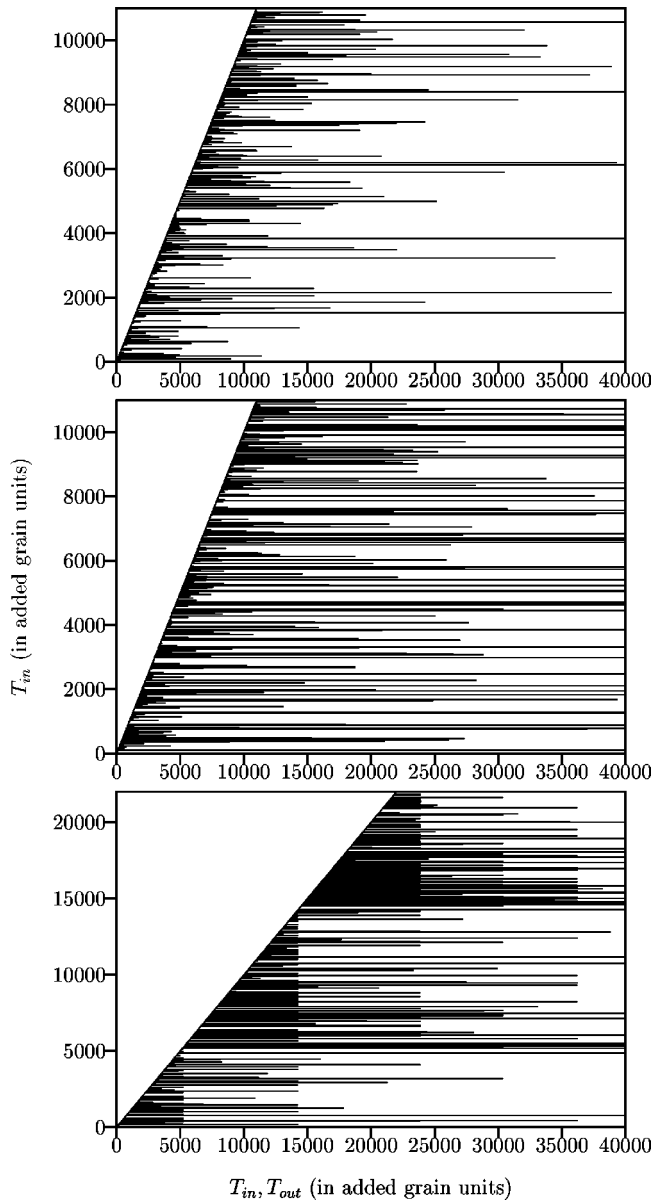
FIG. 1. Sequence of input and output times for different tracers and the three system sizes studied, $L = 15$, 22.5, and 30, from top to bottom. Different tracers are represented by different lines, the starting point is the input time, the end is the output time, and the transit time is represented by the length. Zero time corresponds to the starting of the tracer injection. Time units are set by the driving rate, the window shown in the x axis corresponding to 15.5 h. Notice that for the big pile 1 out of 44 grains is a tracer, whereas for the other two the proportion of tracers is double (2 out of 45); therefore the plots underestimate the difference between both behaviors.

difference between the two smallest piles and the big one. In the first case [Fig. 2(a)] we obtain a power-law distribution of transit times with an exponent close to one, i.e., $D(T,L) \propto 1/T^\alpha$, with $\alpha \approx 1$, ranging from tens of seconds to tens of hours, over more than 3 decades. This is a signature of anomalous diffusion and dispersive transport, and it has been widely studied in amorphous semiconductors, glasses, and many other disordered materials [19].
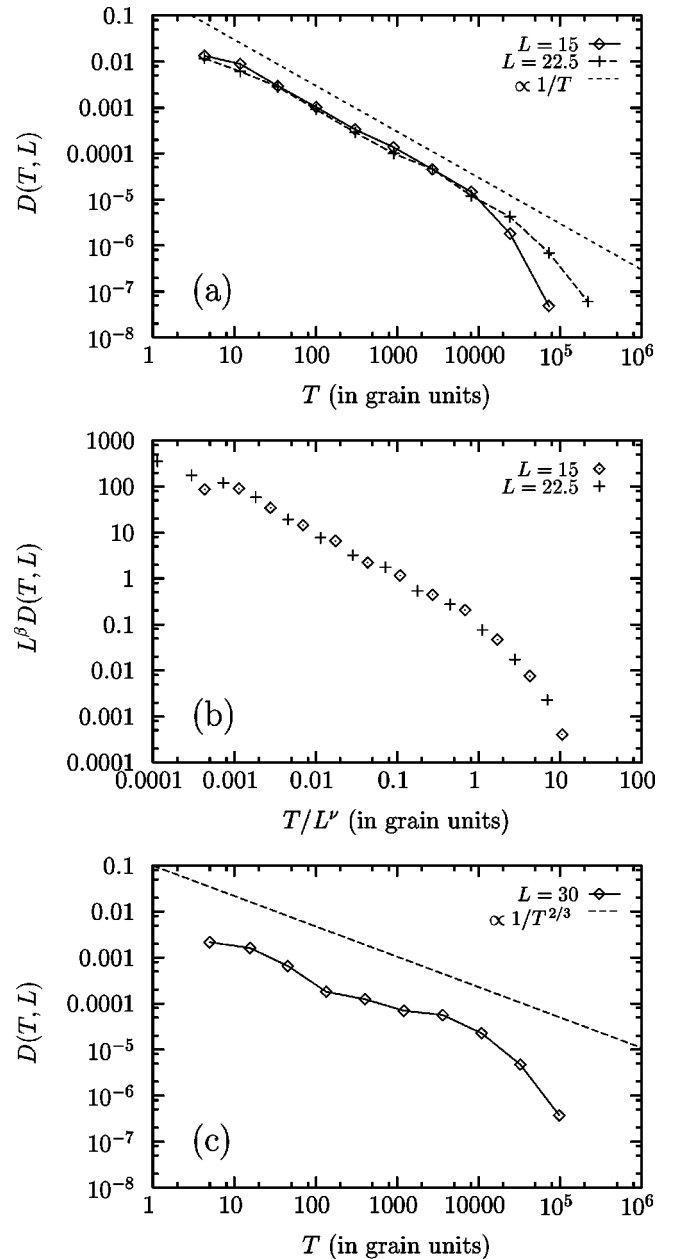

FIG. 2. Probability densities corresponding to the transit times for the system sizes studied, calculated over exponentially increasing intervals of size b^n. As usual, time is measured in terms of added grains and L in terms of grain length. Times smaller than 3 s are not included. (a) The two smallest piles, using $b = 2.5$. The straight line is a decreasing power law with exponent 1. (b) Finite size scaling of the previous distributions, with $b = 2$ now. (c) The big pile, with $b = 2.5$ again. A power law with exponent 2/3 is also shown.

This self-similar behavior is limited by a sharper cutoff for long times, which increases with system size. Finite size scaling, shown in Fig. 2(b) ratifies this, giving the relation $D(T,L) = L^{-\beta} f(T/L^\nu)$ between the distributions for different sizes of the system. The data collapse gives for the exponents $\beta \approx \nu \approx 3.3 \pm 0.5$, in agreement with a well known scaling relation (obtained from the normalization condition), $\beta = \nu$ if $\alpha \lesssim 1$. Also, from these relations it is easy to obtain

that the mean transit time scales as $\langle T(L) \rangle \propto L^\nu$, which is a very fast increase. This procedure is more practical than the direct calculation of $\langle T \rangle$, since it can be used when data for some extreme events are unavailable, as we have for very deeply buried tracers. Notice also that a generalization of the argument of Ref. [5] would give an active zone increasing as $L^{\nu-2}$, so, even in the regime of small avalanches there is an active zone that increases with system size.

However, further increments on the size L completely break this behavior and the previous scaling relations are no longer valid. Indeed, for the big pile we enter into the regime of large relaxation avalanches and the transport properties consequently change. As we see in Fig. 2(c) the transit time distribution becomes even broader as it decays more slowly. A decreasing power law with exponent 2/3 is shown for comparison, although the statistics is not so good as for the small piles, due to the extremely slow decay. We find particularly relevant that despite of the almost periodic ticking of the pile, there is no characteristic scale for the transit times.

IV. CONCLUSIONS AND PERSPECTIVES

As we have seen, three-dimensional piles of elongated particles behave essentially as piles of round particles in 3D, rather than as $1+1$-dimensional rice piles. Therefore the transport properties we have found should apply to any sandpile in 3D [11,12]. It is an open question if with an even more dissipative particle shape, piles in 3D would give scale invariance in the distribution of avalanches. In fact we believe that the use of particles with several ''arms,'' like crosses or 3D crosses, or some kind of sticky particles, would brake the tendency to roll and the results—if this kind of particles could be experimentally handled with success— would be closer to SOC.

However, although we do not see indication of SOC in the distribution of avalanches, we find that the transit time of the grains is power-law distributed, signaling the existence of a scale-free behavior, as in the $1+1$-dimensional SOC rice pile of Refs. [7,5]. Nevertheless it is worth mentioning that the type of distributions that we find are much broader (in the sense that the exponent α is much smaller) than those of the $1+1$-D case.

Big piles, that is, those in the relaxation-oscillation regime, demand a more intensive study. Notice that we have studied the smallest of these big piles, just above the transition, but we have found enormous transit times which provoke serious inconveniences in the experimental realization. Indeed, the fact that the transit time distribution decays more slowly than for the small piles, gives a larger number of tracers in the tail of the distribution, which corresponds to long-living particles. This effect increases dramatically with system size; therefore, much better statistics is necessary for the elucidation of the characteristics of the transport process. We are talking about tens of thousands of tracers, which requires a different procedure than our hand addition and visual control, so man-hour consuming. In consequence, automatization of the tracer recognition process would be highly desiderable.

Finally, none of the sandpile models we know so far is able to reproduce these results; accordingly, it would be of maximum interest to develop models that allow the necessary interplay between theory and experiment to overcome our present knowledge of these media.

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